

MA 311 – Linear Algebra

Homework #1

Due: January 31, 2023

1. Find vectors \mathbf{v} and \mathbf{w} so that

$$\mathbf{v} + \mathbf{w} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \text{and} \quad \mathbf{v} - \mathbf{w} = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}.$$

2. Determine values of α , β , and γ so that $\alpha\mathbf{u} + \beta\mathbf{v} + \gamma\mathbf{w} = \mathbf{b}$ where

$$\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

3. Show that the vector \mathbf{u} is a unit vector for all θ and ϕ where

$$\mathbf{u} = \begin{bmatrix} \cos \theta \cos \phi \\ \sin \theta \cos \phi \\ \sin \phi \end{bmatrix}.$$

4. If $\|\mathbf{v}\| = 5$ and $\|\mathbf{w}\| = 3$, what are the smallest and largest values of $\|\mathbf{v} - \mathbf{w}\|$? What are the smallest and largest values of $\mathbf{v} \cdot \mathbf{w}$?
5. Find four orthogonal unit vectors with each components equal to either $\frac{1}{2}$ or $-\frac{1}{2}$.
6. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

- (b) Solve the following system of equations

$$\begin{aligned} x + 2y + 3z &= 1 \\ y + 4z &= 1 \\ 5x + 6y &= 2 \end{aligned}$$

for x , y , and z .

7. Construct a 3×3 matrix that is not invertible, and explain why it is not invertible.