

MA497 Numerical Analysis

Homework 2

Due: 14 March, 2008

The solution to all problems should be typeset using L^AT_EX, and the L^AT_EX source file should be submitted electronically via email. The submitted source file will be compiled using `pdflatex` and the success of this process will represent the initial assessment step. All source code for numerical algorithms should also be submitted electronically via email.

1. Determine the order of convergence and asymptotic constant of the sequence $\{p_n\}_{n=1}^{\infty}$ where

$$p_n = \frac{1}{e^{e^n}}.$$

2. Use Newton's method to determine the unique real root of $f(x) = x^3 - 2x + 2$. Begin with $p_0 = 0$. Why does this process fail? What provisions can be made to ensure that the iterative process converges to the root of f ? Make these provisions, and approximate the root.
3. Let

$$r(t) = \frac{5t^3 - 6t^2 - 24t}{15t^3 - 30t - 10}$$

be a rational function on the interval $[0, 1]$.

- (a) Determine **the** cubic polynomial $p(t)$ that satisfies the following four properties

- $p(0) = r(0)$
- $p(1) = r(1)$
- $p'(0) = r'(0)$
- $p'(1) = r'(1)$

- (b) By letting $e(t) = r(t) - p(t)$ be the error between $r(t)$ and $p(t)$, it follows that

$$e(t) = -\frac{3t^2(t-1)^2(46t^2 + 11t - 56)}{25(3t^3 - 6t - 2)}$$

and thus

$$e'(t) = -\frac{6t(t-1)(207t^6 - 36t^5 - 774t^4 - 78t^3 + 699t^2 + 89t - 112)}{25(3t^3 - 6t - 2)^2}.$$

What is the largest value of $|e(t)|$ (i.e., what is the largest amount of deviation of $p(t)$ from $r(t)$) on $[0, 1]$?

4. Table 1 shows Iran's annual energy consumption (in quadrillion BTU's) between the years 1981 and 2000. Use a least squares model to estimate their energy consumption in the year 2008. Comment on how well you believe this estimate predicts Iran's actual energy consumption in 2008.

Year	Quadrillion BTU's
1981	1.5
1982	1.7
1983	2.1
1984	2.2
1985	2.3
1986	2.4
1987	2.5
1988	2.7
1989	2.9
1990	3.1

Year	Quadrillion BTU's
1991	3.2
1992	3.3
1993	3.4
1994	3.6
1995	3.8
1996	4.0
1997	4.2
1998	4.5
1999	4.6
2000	4.7

Table 1: Iran's Annual Energy Consumption

5. The function $f(x) = \sin \pi x$ resembles a *quadratic* on $[0, 1]$. Approximate $f(x)$ by some quadratic on $[0, 1]$. Next, sample $f(x)$ at several points in $[0, 1]$ and apply a least squares quadratic approximation to this data to obtain another quadratic approximation. How do these quadratics compare to one another? As the number of sample points increases, what happens to the approximation?