MA497 Numerical Analysis
Homework 1

Due: 7 February, 2008

The solution to all problems should be typeset using LaTeX, and the LaTeX source file should be submitted electronically via email. The submitted source file will be compiled using `pdflatex` and the success of this process will represent the initial assessment step. All source code for numerical algorithms should also be submitted electronically via email.

1. Let $f(x) = 2\sqrt{x} - \cos x$.

   (a) What does the intermediate value theorem imply about f on the interval $[0, 1]$?

   (b) Show that f on $[0, 1]$ satisfies each of the hypotheses of Theorem 2.1 on page 49 of the text.

   (c) Theorem 2.1 recalls that the Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ that converges to a zero p of f, and it implicitly provides an upper bound for the number of iterations needed to guarantee $|p_n - p|$ be within a certain amount. Using this theorem, determine an *a priori* estimate of the number of iterations need to guarantee $|p_n - p|$ be no more than 10^{-4} on $[0, 1]$.

   (d) Write a bisection method algorithm, called `bisection.m`, which implements this problem, and use this algorithm to determine the **ACTUAL** number of iterations N needed to guarantee that

   $$|f(p_N)| \leq 10^{-4}.$$

   How does this number compare to the estimate in part (c)? Explain this discrepancy.

2. (a) Determine a contraction mapping g and corresponding Lipschitz constant k on $[1, 2]$ with a unique fixed point $x \in [1, 2]$ such that $x^3 - x - 1 = 0$.

   (b) With an inital starting point $p_0 = 1$, use Corollary 2.4 on page 59 of the text to estimate the number of iterations required to guarantee the approximate fixed point is accuracate to within 10^{-6}.

   (c) Write a fixed point algorithm, called `fixedpoint.m`, which implements this problem, and determine the **ACTUAL** number of iterations needed to achieve the accuracy in (b). How does this compare to the answer in (b)?