Cubic Bézier Curves

To complete the job of path specification, we need merely explain how to change a segment like '$z_0\{w_0\}$ .. tension α and β ..$\{w_1\}z_1$' into a segment of the form 'z_0 .. controls u and v .. z_1'; i.e., we finally want to know METAFONT's magic recipe for choosing the control points u and v. If $\theta = \arg(w_0/(z_1 - z_0))$ and $\phi = \arg((z_1 - z_0)/w_1)$, the control points are

$$u = z_0 + e^{i\theta}(z_1 - z_0)f(\theta, \phi)/\alpha,$$

$$v = z_1 - e^{-i\phi}(z_1 - z_0)f(\phi, \theta)/\beta,$$

where $f(\theta, \phi)$ is another formula due to John Hobby:

$$f(\theta, \phi) = \frac{2 + \sqrt{2}(\sin\theta - \frac{1}{16}\sin\phi)(\sin\phi - \frac{1}{16}\sin\theta)(\cos\theta - \cos\phi)}{3(1 + \frac{1}{2}(\sqrt{5} - 1)\cos\theta + \frac{1}{2}(3 - \sqrt{5})\cos\phi)}.$$

It should be noted that pairs are considered to be complex numbers, and that '..' is actually a synonym for '.. tension 1 and 1 ..'. Notice that the function

$$z(t) = (1 - t)^3 z_0 + 3(1 - t)^2 t\, u + 3(1 - t)t^2 v + t^3 z_1$$

is the cubic Bézier curve with knots z_0 and z_1 and control points u and v.